Mathematics Teachers' Teaching Practices in Relation to Textbooks: Exploring Praxeologies

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Abstract: *In this article, we explore affordances of adopting the framework of praxeology by Chevallard in the analysis of mathematics classroom communication in relation to the communication in a textbook. While adopting praxeology, we carried out detailed analysis of communication in both classroom data and textbooks. The construed praxeologies describe the organisation of knowledge expressed for the same type of task in both classroom and textbook. The praxeologies were compared, with specific attention to the teacher's practice. This analysis illuminates how teachers' practices, realised in classroom communication, may be compared to other texts describing the same topics, with a focus on procedures, explanations, theoretical aspects, et cetera. Hence, praxeology as a framework enabled an analytical structuring of classroom and textbook communication, and consequently a systematic comparison. In other studies about the use of mathematics textbooks the teaching frequently is categorised as regulated by the textbook, and in this article, we problematize this. The teaching practice was, in fact,closely related to the textbook when comparing exercises and procedures, but when specifically examining the explanations of concepts, it became possible to discern how the teaching practice differed from the textbook.*

Keywords: Mathematics teacher education; textbooks; teaching practices; praxeology

Introduction

Before a mathematical concept is taught in a classroom, definitions and descriptions of this concept undergo transformations to be teachable in the particular classroom. These transformations are executed in teachers' classroom practices and also by other actors in the sphere of those who think about teaching (Chevallard & Bosch, 2014). Such transformations may be labelled didactic transposition (Chevallard, 2006). Textbook authors are examples of actors, and textbooks are commonly texts with examples of transformations of mathematics for the purpose of mathematics teaching. In this study, the textbook is explored from a teacher and teaching viewpoint. Consequently, we view the textbook as one description of how mathematics could be taught. More specifically, this study is focused on the development of a methodology for investigating the junction between a teacher's classroom practice in relation to how the textbook ´deals with´ the same concepts.

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Teachers' teaching practices have been studied from individual perspectives, focusing on for example, teachers' decision making about their teaching practices (Bishop & Whitfield, 1972; Borko, Roberts, & Shavelson, 2008; Schoenfeld, 2011), or teachers' individual knowledge (Shulman, 1987). Common to studies with an individual perspective is the focus of the individual without any or with little account for the social environment in this individual's actions (Lerman, 2000). Lerman suggests a turn where the individual is not only studied in relation to practice, but within practice. This could for example be studies of teaching practices within an educational system, where the construction of pedagogic discourses in different levels of this system is examined (Tsatsaroni, Ravanis, & Falaga, 2003), or how different levels of an educational system participate in determining possible teaching practice (Artigue & Winsløw, 2010). From a socio-political perspective, Valero (2004) criticizes socio-cultural studies where researchers 'settle with' an assumption that students and teachers are social beings. She suggests more interest in the very reasons for how practices "are valued as the 'right' way of teaching" (p. 16). This would imply that understanding the contexts surrounding a teaching practice as intertwined, would be powerful compared to understanding these contexts separately. To focus on both the micro perspective of the individual teacher and the macro perspective of the surrounding contexts have been described as a practice turn.. In such studies individual actors are regarded as significant, since their practical skills, when they deal with the constraints from surrounding contexts, make a difference (Whittington, 2006).

Frequently, the mathematics textbook is described as being a central resource for mathematics teaching (e.g., Pepin & Haggerty, 2003). Textbooks can inform the organisation of mathematics teaching, for example when teachers base their teaching on the outline presented in the textbook, while working systematically through the whole content of the textbook (Barr, 1988). Textbooks are also described as a translation of policy into practice (Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002), or as the authorities' prescription about what mathematics to teach and how to do so (Barbé, Bosch, Espinoza, & Gascón, 2005). The influence of textbooks on mathematics teaching has been described both as indirect, when the textbook facilitates teaching for teachers (Pepin & Haggerty, 2003), and direct, when teachers choose to adopt textbooks exclusively without additional materials. Teachers can also choose additional content, or teach using a different structure from the textbook (Sosniak & Stodolsky, 1993), or take more factors into account, for example assessment data, when preparing mathematics teaching (Sullivan, Clarke, Clarke, Farrell, & Gerrard, 2013). The mathematics textbook is throughout these studies described as an artefact that to some extent reflects teaching practice. In this study, the mathematics textbook is examined as one example of the broader context to a teacher's practice. A literature search on the words *teaching*, *teacher*, *mathematics*, and *textbook*, produced many hits. Included below are a number of international studies and reviews, with a main focus on the methodologies adopted. Studies from a Nordic and Swedish perspective serve as a description of a cultural influence on mathematics education (Andrews 2016b) in relation to the interest of this study.

In textbook research there are different methods adopted for different foci. When the focus is on textbook use, the teacher is usually in the centre of the research, and the use of textbooks is often studied through interviews. Teachers may describe how they use their teacher guides (Ahl, Gunnarsdóttir, Koljonen, & Pálsdóttir, 2015), or describe how they feel forced to rely on textbooks due to time pressure (Pepin & Haggarty, 2001), but also how they use theory, examples, and exercises to influence their teaching (Viholainen, Partanen, Piiroinen, Asikainen, & Hirvonen, 2015). Questionnaires have been used to study how teachers claim to use textbooks in a broader sense (Lepik, Grevholm, & Viholainen, 2015), describing mathematics textbooks as "a primary information source for teachers in deciding how to present the content" (p. 132). In these studies, it is the teachers' description of textbook use, that grounds the conclusions. When the textbook is examined, the study sometimes concerns the textbook as a text. For example Österholm and Bergqvist (2013), carried out linguistic studies on mathematics textbooks, showing that the text is more compact, complex, and technical compared to books from other subjects. Another example is Herbel-Eisenmann (2007) who performed discourse analysis on mathematics textbooks, studying the ideological goals underlying certain curriculum materials. The contents of the textbooks may be studied, for example in a comparison between English and Swedish textbooks (Löwenhielm, Marschall, Sayers, & Andrews, 2017). In that study, the English textbooks were shown to emphasize procedures, while the Swedish textbooks emphasized different representations of numbers and conceptual matters.

In a study on both textbooks and teacher practice, Haggarty and Pepin (2002), compared practices in the contexts of France, Germany, and England. The authors adopted a scheme, while drawing on diverse literature, to study how mathematics was regarded in the different countries' textbooks. Grave and Pepin (2015) showed how teachers' use of textbooks became different in different teaching situations. Combining textbook analysis and observations with analysis of policy documents, Pepin, Gueudet, and Trouche (2013) showed how educational and cultural traditions "weave" their way from policy level through textbooks into the classroom. Drawing from the coding manual of TIMSS 1999 video study, Johansson (2006) described the direct presence of mathematics textbooks in the mathematics classroom, not only in the students' work but also in the teachers' explanations or examples. Studying the communication in the text, and describing and explaining the different topics (the didactical layer) of a textbook, Jablonka, Ashjari, and Bergsten (2016) adopted concepts by Bernstein (2000) to study the level of classificatory principles of the textbook in relation to what the students recognised as legitimate mathematics. Furthermore, the study showed how students' recognition of the classificatory principles of the mathematics pedagogic discourse was linked to their success in examinations. These studies imply in different ways that mathematics textbooks carry privileged images of mathematics, which teachers may include in their negotiations about what to teach and how.

Praxeology by Chevallard (2006) has been adopted as a methodology for the study of how mathematics is organized in different contexts where knowledge, seen as human activity, is described to consist of both praxis (knowing how) and logos (knowing why). Larson and

Bergsten (2013), for example, compared changes in classroom communication when students transferred from compulsory to upper secondary school in Sweden. They also showed how the instructional language differed in terms of praxis (teacher telling students how to do things) and logos (teacher explaining why). Vendiera-Marechal (2011) focused especially on praxis, the types of tasks and the procedures described to solve these tasks, when comparing teaching practices in different classrooms. She illuminated how praxeologies depend on constraints in the classroom's broader context, for example, how the mathematics textbook in use affects mathematics teaching. Wijayanti and Winsløw (2017) also focused on tasks and techniques when they show how praxeological reference models can be used to analyse the mathematical content in textbooks. They suggest that this may be complemented by a discussion of the discursive environment in the textbooks. Klisinska (2009) describes the discursive environment of scholarly mathematics to be a functional specialised language system for mathematicians, where concepts are well defined, and with deductive forms of arguments. The discursive environment of school mathematics on the other hand, is described to be less formal for readers with less specialised knowledge, and where concepts are not defined but taken for granted.

In the Swedish context, where this study was undertaken, textbooks have been found to dominate teaching practices (Johansson, 2006; Skolverket, 2014). As a consequence of this dominance, much of the curriculum is transposed to classroom practice through textbooks (Jablonka & Johansson, 2010). It is, however, not enough only to say that textbooks dominate classroom practice. Rather, there are more nuances to the use of textbook, as shown by Boistrup (2017) in relation to teacher-student communications showing traces of textbook influence. The dominance of textbooks in mathematics classrooms is, however, frequently critiqued in Swedish public debate, particularly from the perspective of school authorities such as the Swedish Schools Inspectorate (Skolinspektionen, 2009). It has been shown that students worked on exercises from the textbook during mathematics lessons for about 70% of the time (Bergqvist et al., 2010). As a consequence, textbooks substantially affect what Swedish students are occupied with in the mathematics classroom. As reported on from different contexts, both national and international, the use of mathematics textbooks as a central resource in relation to teachers' teaching practices becomes essential to study, and methodologies for such studies need to be developed.

We assume that teachers' teaching practices are enacted in relation to sources external to direct classroom practice. Additionally, we view textbooks as such a source. Through comparisons of classroom observations and textbooks, it is possible to construe transformations of mathematical concepts in terms of what is communicated and how. In the present study, the mathematics textbook is viewed as a text offering a set of exercises, procedures, and explanations. In order to understand teachers' teaching practices in relation to such a text, we needed a framework.

The aim of this study was to explore how praxeology can be used in a methodology where mathematics teaching practice and textbooks are emphasized as two interrelated contexts. At the junction of studies focusing on macro-sociological issues and studies focusing on classroom practices or textbooks, we aimed to fill a gap when studying the micro-sociological layers close to classroom practice, here exemplified by a textbook. More specifically, we aimed at studying the particularities of mathematics instruction in a classroom in relation to instructions in mathematics textbooks, while adopting praxeology as a framework for comparison. In this comparison, we wanted to explore how a praxeology can be used to compare a mathematics teacher's practice, as observed in the classroom, with the practice, as inferred from the textbook.

Anthropological Theory of Didactics

In order to study a teacher's teaching practice in relation to the surrounding context (in the case of this article exemplified by the textbook), we have explored praxeology by Chevallard (2006). Praxeology is part of the Anthropological Theory of Didactics, ATD, which offers a framework to investigate human action in relation to social institutions. In this study, the human action examined is a teacher's teaching practice, studied in relation to the curriculum as interpreted from textbooks. ATD has been used to study didactic phenomena located in different social practices (Chevallard & Sensevy, 2014). When addressing relationships between these practices, ATD enables the description of "didactic transposition," suggesting that knowledge expected to be taught in schools is transposed to be teachable in specific classrooms (Chevallard, 2007). That is;

> The process of didactic transposition refers to the transformations an object or a body of knowledge undergoes from the moment it is produced, put into use, selected, and designed to be taught until it is actually taught in a given educational institution (Chevallard & Bosch, 2014, p. 170).

Transpositions occur from one social institution to another, for example, from national curriculum to textbooks (external transposition), in the sense that when knowledge developed in one institution is needed in another it has to be transformed to adapt to the new institutional setting (Chevallard, 1991; Chevallard & Bosch, 2014; Østergaard, 2013)[2]. Barbé et al. (2005) together with Bosch and Gascón (2006) claim that teaching processes cannot be interpreted without taking the processes of didactic transposition into account, where practices in school reconstruct mathematics that originates from institutions that produce mathematical knowledge (internal transposition). This is in line with how we position our study, with an interest in exploring how praxeologies may be used to study a teacher's teaching practice in relation to textbooks. This entails the study of didactic transpositions from a mathematics textbook to the mathematics classroom.

A praxeology (figure 1) is how ATD describes knowledge, in the sense of human action (Chevallard & Sensevy, 2014). Through a praxeology, it is also possible to visualise how mathematical content is taught (Barbé et al., 2005; De Vleeschouwer, 2010; Winsløw, Barquero,

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[2] The original reference, Chevallard, Y. (1985). *La transposition didactique* Grenoble: La pensée sauvage, was not available to us which is why the Spanish translation is referred to in the article.

De Vleeschouwer, & Hardy, 2014). A specific praxeology constitutes a framework for a specific use (Jablonka & Bergsten, 2010), for example, angles, fractions or volume, in grade five, as in this article. This framework is built on two main components, praxis (know how) and logos (know why) (see Figure 1).

Praxis, the know-how	
Task	*Technique*
Logos, the know-why	
Technology – rationale for techniques	*Theory* – overall rationale

Figure. 1. Praxeology.

Praxis is described as including *tasks* that are to be solved using certain *techniques* (see Figure 1), in the sense of procedures that will solve the tasks. For explaining *why* the techniques apply there is a logos, a *technology*, with explanations and arguments used to back up the technique (e.g., proofs and properties) (Østergaard, 2013). Additionally, for putting the technology into a wider context of meanings, there is a *theory*. The theory explains, justifies or produces a technology, while establishing a deeper level of justification of the practice (Barbé et al., 2005; Bosch & Gascón, 2006; Chevallard, 1998). Moreover, theory creates a discursive environment for praxis (Straehler-Pohl & Gellert, 2013), where notions, properties, and relations establish technologies, techniques, and tasks (Gellert, Barbé, & Espinoza, 2013). In the language of scholarly mathematics, theory could, for example consist of definitions and axioms (Østergaard, 2013). Similar to Miyakawa and Winsløw (2013) and their studies of teacher knowledge, we have analytically combined technology and theory. Moreover, we see the analytical 'space' of technology/theory as a continuum from a school mathematical language to the language of scholarly mathematics. This is similar to how Straehler Pohl and Gellert (2013) describe it, with a continuum from theoretical explanations with the language use of definitions and axioms at one end, and simple explanations weakly backing up a technique at the other.

Looking at a classroom practice, praxis may concern the knowing of techniques, for example how to set up an equation, how to solve a problem, or how to divide a fraction. Logos may be construed from how a mathematical statement is described and justified (Barbé et al., 2005), for example an explanation of why a problem is solved in a certain way, or why the right angle may be used as a reference when you measure angles with a protractor. A praxeology is socially situated, and, consequently, a praxeology concerning for example algebra is not the same in upper secondary as it is at the university (Winsløw et al., 2014). Bosch and Gascón (2006) show how the institutional and social contexts can constrain a praxeology, and how one problem is to know from where in the broader context such constraints are derived. In this article, we illuminate how praxeology enabled us not only to study *whether* the textbook constrains teachers' teaching practices, but, from a broader perspective, to understand *how* such relationships between textbooks and classroom practices may be constituted.

Methodology

We view the present study as a telling, instrumental case study. A telling case study is a typical form of case study, described to offer insights previously unavailable or hidden (Andrews, 2016a; Mitchell, 1984). Stake (1995) describes an instrumental case study as illuminating research questions rather than the case itself. Using a single case facilitates a more thorough analysis of the case within its social setting (Hammersley & Gomm, 2009), with more depth possible than when using multiple cases (Donmoyer, 2009; Gerring, 2006). In this case study, we searched for a way to study a textbook's and a teacher's classroom praxeologies in relation to each other, which makes this an instrumental case, and more specifically a telling case.

Methods for Data Collection

We collected data from three different grade five mathematics lessons given by the same teacher. All written material used in the lessons were collected, for example textbooks, assignments, posters, and pictures of drawings on the whiteboard. Data collection was made by both video and audio recordings. One camera facing the teacher captured her whole class teaching. One microphone carried by the teacher captured her spoken communication, and one extra microphone captured the sound from the back of the classroom.

The data analysed in this study consisted of multimodal transcripts, meaning not only spoken or written words were included, but also gestures and pictures (Kress, 2015). We included transcripts of the first part of three lessons, where three different concepts - fractions, volume, and angles - were introduced for the first time. The introductions to the same concepts in the chosen textbook were also included in the data set. In her teaching of angles, the teacher chose to use a complementary textbook (normally used in grade seven) which is why we analysed the communication from this textbook. When teaching fractions, the teacher used the regular textbook which was chosen by the teachers in the school, and used by all teachers teaching grade five in the same school. When teaching the topic of volume, the teacher used sources other than the textbook. The other teachers teaching grade five did, however, use the regular textbook. Because the students were assessed according to the suggested tests from this book, we decided to analyse the regular textbook even if the teacher had used other sources.

Method of Analysis

We construed the mathematical knowledge intended to be taught (textbook) and actually taught (classroom practice) as praxeologies (Chevallard, 2006). Each praxeology reflects how a specific topic's content had been transposed into this context. For the analysis of relations between a teacher's teaching practice and the communication in the mathematics textbook, we compared the praxeologies of each context, classroom and textbook. The construed praxeologies, which concern mathematical content (angles, volume, and fractions) in grade five, can be seen as 'point praxeologies' (Winsløw et al., 2014), each concerning a single type of task.

In this study, the starting point for the construction of each praxeology was a specific type of task. This does, however, not mean that tasks always were the starting point in either the teacher's or the textbook's communication. In the examples from both practices in this article, explanations preceded presentations of exercises to students. Hence, we made the analytical choice to construe the praxeologies based on the types of tasks, in order to enable a comparison of praxeologies. All the exercises in the textbook-sections included in the study were categorised into different types, and where possible we identified a similar type of task from the classroom practice. In the first section on angles, as an example, most exercises consisted of a straightforward instruction on how to measure different angles. How to measure an angle was thus one type of task. In other exercises, the students were asked to draw angles of a specific size ('how to draw an angle'). For the task type 'how to compare angles', we collected all exercises where the students were asked to decide which angles were acute, right, or obtuse. Each type of task gave rise to one construed praxeology. In total, we identified 15 types of tasks, and consequently 15 different praxeologies, from both classroom and textbook communication. The inferred techniques, technologies, and theory in a construed praxeology, all referred to exercises of the same type of task.

The techniques interpreted for each type of task were the communicated procedure(s) that could help in solving such a task. In the textbook there was, as an example, a detailed description of how to draw an angle using a protractor. This was inferred as a technique for the type of task, 'how to draw an angle'. The technology for each type of task was the communicated explanations or descriptions of properties that could justify the presented procedure(s). If there were no procedures or explanations, these boxes were left blank. As an example, in the angle section we inferred the properties of right, acute, and obtuse angles as technology/theory. In relation to the continuum between theory and technology (see figure 2), this explanation would be inferred as technology with a modest theoretical depth. The reason for this is that properties were described, but the description was not very close to any definition or axiom; rather it was used to justify the presented technique. We analysed the techniques and technologies/theories adopting a multimodal approach (van Leeuwen, 2005), meaning that a definition could be constituted by a gesture and/or a picture, as well as with spoken communication.

Textbook praxis	
Task	*Technique*
A type of task	How to solve the task
Textbook logos	
Technology/Theory	
Why this task is solved this way, the rationale for the technique including both explanations and definitions.	

Classroom praxis	
Task	*Technique*
A type of task	How to solve the task
Classroom logos	
Technology/Theory	
Why this task is solved this way, the rationale for the technique including both explanations and definitions.	

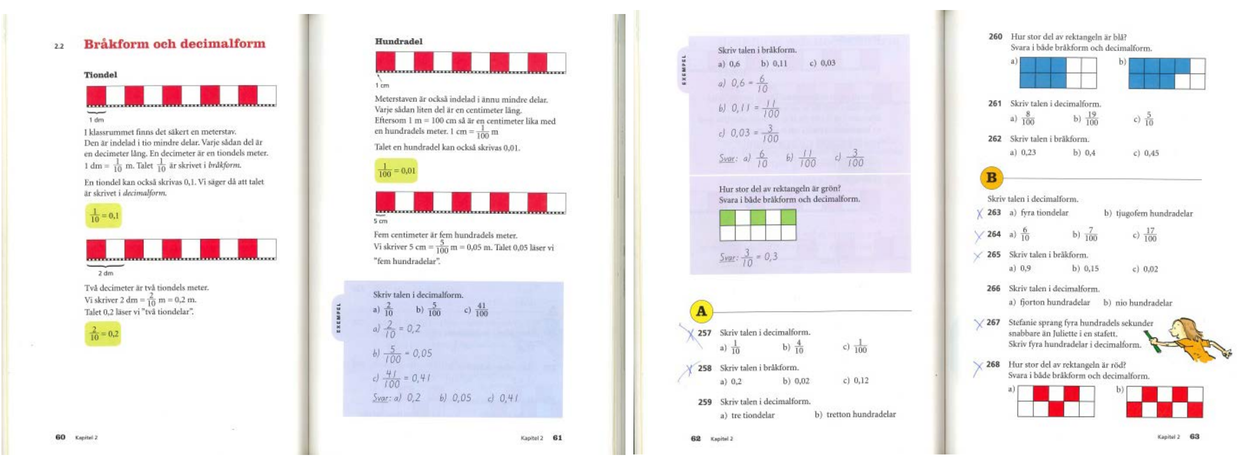
Figure 2. Praxeologies of textbook and classroom practice.

Findings

Our findings consist of descriptions of our analysis of praxeologies. Hereby the adopted methodology is illuminated through the presentation of two praxeologies, the textbook and classroom practice. Our description of the construed praxeologies follows the structure of Figure 2: type of task, techniques (procedures), and technology (explanations)/theory, for each of the practices in the textbook and in teaching. In a summary of the findings, the construed praxeology is described through a figure where the model from Figure 2 is adopted (see Figure 9). Before describing our findings, we introduce the local context where the data collection for this study was performed.

Local Context

With regards to the textbook context, we observed a teacher, here called Mary, choosing two different textbooks, one normally written for the fifth grade (Undvall, Forsberg, & Melin, 2006, illustrations Johan Unenge) and one textbook written for seventh grade (Undvall, Olofsson, & Forsberg, 2001). As described earlier, the grade five book was chosen by the school and it was used in the other grade five classes as well. The two textbooks shared the same structure for any given topic: first a few pages with explanations, instructions and examples, followed by a few pages with exercises mirroring the examples presented in the first pages (see Figure 3).



2.2 **Bråkform och decimalform**

Tiondel

1 dm

I klassrummet finns det säkert en meterstav. Den är indelad i tio mindre delar. Varje sådan del är en decimeter lång. En decimeter är en tiondels meter. 1 dm = $\frac{1}{10}$ m. Talet $\frac{1}{10}$ är skrivet i *bråkform*.

En tiondel kan också skrivas 0,1. Vi säger då att talet är skrivet i *decimalform*.

$\frac{1}{10} = 0,1$

2 dm

Två decimeter är två tiondels meter. Vi skriver 2 dm = $\frac{2}{10}$ m = 0,2 m. Talet 0,2 läser vi "två tiondelar".

$\frac{2}{10} = 0,2$

60 Kapitel 2

Hundradel

1 cm

Meterstaven är också indelad i ännu mindre delar. Varje sådan liten del är en centimeter lång. Eftersom 1 m = 100 cm så är en centimeter lika med en hundradels meter. 1 cm = $\frac{1}{100}$ m

Talet en hundradel kan också skrivas 0,01.

$\frac{1}{100} = 0,01$

5 cm

Fem centimeter är fem hundradels meter. Vi skriver 5 cm = $\frac{5}{100}$ m = 0,05 m. Talet 0,05 läser vi "fem hundradelar".

EXEMPEL

Skriv talen i decimalform.

a) $\frac{2}{10}$ b) $\frac{5}{100}$ c) $\frac{41}{100}$

a) $\frac{2}{10} = 0,2$

b) $\frac{5}{100} = 0,05$

c) $\frac{41}{100} = 0,41$

Svar: a) 0,2 b) 0,05 c) 0,41

Kapitel 2 61

EXEMPEL

Skriv talen i bråkform.

a) 0,6 b) 0,11 c) 0,03

a) $0,6 = \frac{6}{10}$

b) $0,11 = \frac{11}{100}$

c) $0,03 = \frac{3}{100}$

Svar: a) $\frac{6}{10}$ b) $\frac{11}{100}$ c) $\frac{3}{100}$

Hur stor del av rektangeln är grön? Svara i både bråkform och decimalform.

Svar: $\frac{3}{10} = 0,3$

A

257 Skriv talen i decimalform.
a) $\frac{1}{10}$ b) $\frac{4}{10}$ c) $\frac{1}{100}$

258 Skriv talen i bråkform.
a) 0,2 b) 0,02 c) 0,12

259 Skriv talen i decimalform.
a) tre tiondelar b) tretton hundradelar

62 Kapitel 2

260 Hur stor del av rektangeln är blå? Svara i både bråkform och decimalform.
a) b)

261 Skriv talen i decimalform.
a) $\frac{8}{100}$ b) $\frac{19}{100}$ c) $\frac{5}{10}$

262 Skriv talen i bråkform.
a) 0,23 b) 0,4 c) 0,45

B

Skriv talen i decimalform.

263 a) fyra tiondelar b) tjugofem hundradelar

264 a) $\frac{6}{10}$ b) $\frac{7}{100}$ c) $\frac{17}{100}$

265 Skriv talen i bråkform.
a) 0,9 b) 0,15 c) 0,02

266 Skriv talen i decimalform.
a) fjorton hundradelar b) nio hundradelar

267 Stefanie sprang fyra hundradels sekunder snabbare än Juliette i en stafett. Skriv fyra hundradelar i decimalform.

268 Hur stor del av rektangeln är röd? Svara i både bråkform och decimalform.
a) b)

Kapitel 2 63

Figure 3. Four pages from the fifth grade mathematics textbook, Matematikboken (Undvall et al., 2006, illustrations Johan Unenge).

The textbooks sometimes included explanations of the content matter, often to introduce something new. There were always examples showing how an exercise could/should be solved, often accompanied by comments on the solutions. Frequently, yellow boxes, which highlighted essential information, occurred (see Figure 10).

With regards to *classroom context*, the observed lessons also followed a pattern. Mary began the lesson with a 15-20 minutes introduction to the content matter. This introduction was followed by student work, often with exercises from a textbook, but there could also be exercises from other sources. The communication in the introduction consisted of questions from both Mary and her students, as well as Mary telling the students about the mathematical concepts.

Comparing praxeologies

We describe our comparison between the textbook and Mary's classroom practice in relation to task, technique and technology/theory. We offer several data excerpts in order to make our interpretations transparent.

Tasks

In the construed praxeologies, the types of tasks in the textbook were the same as the exercises exploited during the lesson. In the following, we use the teaching of angles as the overall example of such a praxeology.

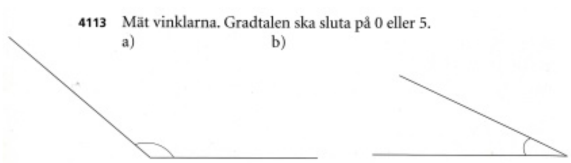


Figure 4. Exercise from the textbook.

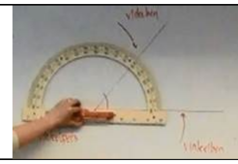
We identified three different types of tasks in the angle section of the textbook: how to measure an angle (see Figure 4), how to draw an angle, and how to compare angles. After three pages in the textbook with these kinds of exercises, there was another sub-section, which focused on characteristics of different triangles. Finally, there were three pages of exercises on measuring angles in triangles, determining an angle in a triangle with two known angles, and determining angles in different types of triangles using the facts known about them. In the following we describe the techniques we inferred to be applied to the type of task 'how to measure an angle.' We construed two praxeologies, one from the communication in the textbook and one from the classroom communication. Both praxeologies share the same types of tasks.

Techniques

The techniques concerning the content of angles were partly similar in both the textbook and the lesson.

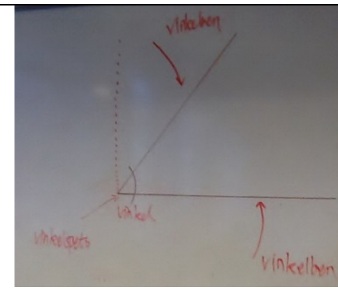
Figure 5. Text in English: Angles can be measured with the help of a protractor. With the help of the protractor we can see that the measured angle is 60°.

The textbook stated that an acute angle is smaller than 90° and an obtuse larger than 90° (Figure 5). Later, the textbook showed an obtuse angle being measured, together with the written information that "Measuring an obtuse angle you use the scale on the protractor graded between 90° and 180°." We interpreted this as the technique in the textbook, in the sense of how to use the protractor. In excerpt 1, we exemplify data from which the technique in the classroom communication was interpreted.

Time	Speech	Actions	Whiteboard
00:24:08	Mary: Let's place it here	Mary places the protractor against the drawn angle.	

Excerpt 1. Mary showed the students how to measure an angle using a protractor.

The technique we interpreted from Mary's teaching practice (Excerpt 1) was more or less mirroring the textbook. In the textbook, the measuring of an angle was described in a picture (see Figure 5), and during the lesson the same was described in a sequence where Mary placed the protractor and read the angle. Like the textbook, Mary also addressed the question about how to use the two different scales on the protractor, which we can see in Excerpt 2.

Time	Speech	Actions	Whiteboard
00:24:56	Mary: Now we draw a help-line	Mary draws a dotted line that makes a right angle together with the horizontal angular leg.	

Excerpt 2. Mary explained how to use the two scales on a protractor.

In Excerpt 2, Mary used a dotted helpline, to mark where a right angle could be drawn in relation to the angle they were about to measure. Mary also showed the students how they could estimate the size of the angle before measuring and use this to decide which scale on the protractor to use (Excerpt 3).

Time	Speech	Actions	Whiteboard
00:25:57	Mary: That is why I wanted you to, already before you measure, decide if it is more or less than 90, because you know that already when you look at it, if it is not very, very close to 90.		
00:26:07	Mary: But if you answer that this angle is 130 after you placed the protractor. Then you must react, no! It can’t be 130, I said before that it was an acute angle.	Mary points at the angle on the white board. Mary holds up the protractor	

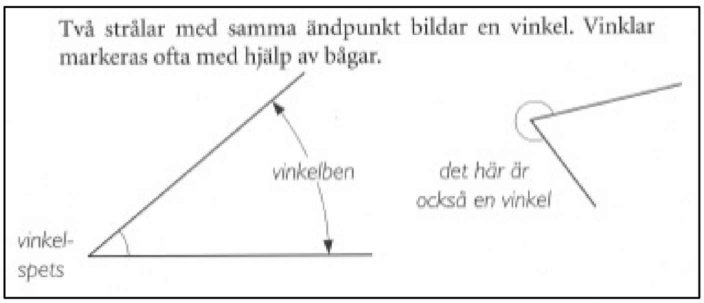
Excerpt 3. Mary continued to explain how to use the two scales on a protractor.

From Excerpt 3, we interpreted a technique, where the angle was first compared to a straight angle, followed by the estimation which was used to decide which scale on the protractor to read. We construed this technique as different from the one in the textbook where it was stated that the scale from 90 to 180 is to be used for obtuse angles, while Mary showed the students how to estimate the size of angles and how to use this estimation to decide which scale to read.

Technology and Theory

As described earlier, we chose to see technology and theory not as two separate categories, but as a continuum from a technical school mathematical language in the explanations of techniques towards a language use closer to scholarly mathematical definitions, proofs, and axioms. The technologies/theories differed to some extent between the classroom and the textbook. We inferred technology/theory from the communication in the textbook with descriptions of the mathematical concepts and how to work with them. From the classroom communication, we construed technology/theory from the teacher-student interaction and the explanations we inferred from both Mary's questions and descriptions. We could infer explanations from Mary's questions in the sense that a question could direct the students' attention to mathematical structures and the like. Technologies/theories were in this case interpreted to *explain* the techniques for how to measure an angle and here we also included definitions and descriptions of properties. The latter is relevant since how an angle is defined not only grounds the use of words for angles, but also the description of *what* it is, that is measured.

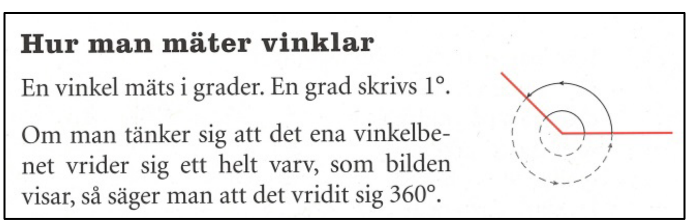
The textbook defined an angle as two rays meeting in a common starting point. To this



definition, a technical description of the different parts of an angle was added (Figure 6).

Figure 6. Definition of an angle from the textbook.

Further down, on the same page, there was a brief reference to the fact that an angle represents a rotation, in the beginning of the description of how to measure angles ("If we imagine that one



angular leg rotates a whole lap, as the picture show, you say that it has rotated 360°", Figure 7).

Figure 7. First paragraph in the description of how to measure an angle.

Even if this could be interpreted as close to a definition of an angle, the emphasis on the parts of the angle and the very brief reference to rotation gave the interpretation of technical language, which we inferred to be closer to technique than theory. The way this was communicated could be read in terms of how to name the parts of the angle rather than as a definition of an angle.

Within Mary's descriptions of angles, we identified definitions, for example a few minutes into the lesson, when Mary responded to a question about why angles are marked with a bow.

Time	Speech	Actions	Whiteboard
00:04:38	Mary: It is to show that two angular legs, as they are called, two angu-, yes angular legs meet in an angle that you mark either with an arc.	Mary moves her hands closer to each other in a movement showing how two angular legs meet in a point.	

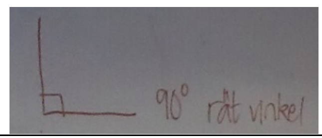
Excerpt 4. Mary explained what an angle is.

We inferred explanations of the properties of angles from both textbook and lesson. They differed in the description of the angular legs, which the textbook referred to as rays and which would be a more mathematically correct description. A similarity between the explanations was the emphasis on technical aspects: The different parts of the angle were mentioned in both explanations. This emphasis comes close to techniques of drawing and recognising angles. Since these were used as explanations in the communication, we still inferred this as technology, however, close to technique. Mary's definition, is circular if we view only the spoken words, since she defines an angle as an angle. However, her gestures add a dimension to the definition. When she moved her hands, she showed both how the angular legs meet to form the angle, as well as the rotation of which the angle is a measure. We inferred this acted rotation as a notion that could be said to explain the technique of measuring angles, and consequently it was inferred as a technology.

The second explanation we interpreted as a rationale for how an angle is measured, were the definitions of acute, straight, and obtuse angles. In the textbook (Figure 8), there was a figure showing these three different angles with a list, where an acute angle was described to be smaller than 90°, a straight angle equal to 90° and an obtuse angle as bigger than 90°. This was interpreted to be a rationale for the technique to measure angles, since there was a statement about how to measure obtuse angles. This is a description of properties which positions it clearly as technology, however, in a school mathematical language.

Figure 8. The textbook's definition of different angles.

In the lesson, we also interpreted definitions of right, acute, and obtuse angles. Mary asked the class what they knew about angles, with the consequence that obtuse, acute, and right angles, were defined as students brought them up. First, Sebastian defined the right angle, as we can see in Excerpt 5.

Time	Speech	Actions	Whiteboard
00:02:18	Sebastian: A right angle, like this /…/	Mary shows a right angle between two fingers (mirroring a gesture Sebastian made, however not on camera).	
00:02:26	Mary: You drew, in the air, that a 90 degree angle looks like this.	Mary draws a right angle, she also writes 90 and right angle.	

Excerpt 5. Mary defined a right angle together with Sebastian.

In Excerpt 5, Sebastian and Mary used gestures and pictures to define a right angle as a 90-degree angle. The definition of an acute angle was identified a few minutes later, in the answer to Gunnar's question about how many degrees are acute and obtuse angles. They have just discussed the size of right angles and Gunnar wants to connect the acute and obtuse angles to a specific value, like the right angle, which has the value 90.

Time	Speech	Actions	Whiteboard
00:04:52	Gunnar: How many degrees is an acute and an obtuse angle, I mean you say that a right angle is 90 degrees, but is there any, like this, an acute angle…		
00:05:06	Mary: Well, yes, an acute angle can go from 89.999999 really many nines and all the way to zero.	Mary shows with a movement of her finger from the dotted line, marking a right angle, how an acute angle can be any angle from a right angle down to the other angular leg, which would make the angle zero.	

Excerpt 6. Mary defined an acute angle.

We interpreted the definition in Excerpt 6 to be different from the textbook's statement that acute angles are smaller than 90 degrees. The content is the same, but in her definition Mary chose to describe the notion of limits for acute angles, that there is a range for the acute angles. The same kind of difference between textbook and lesson was interpreted from Mary's definition of the obtuse angle. If, like Gunnar, one wants to have a specific value for the acute and obtuse angle, the textbook's explanation did not show enough information for a student to see that there can be infinite number of acute and obtuse angles, respectively. Mary's explanation, thanks to her gestures, and how she talks about the scope for acute angles, describes this so that the possibilities for different acute and obtuse angles become clear.

We inferred Mary's descriptions of the angle as being a measurement of a rotation, in the sense that there is an infinite number of acute and obtuse angles, to draw from notions and definitions of scholarly mathematics. As a comparison, we inferred the textbook's technical description of the parts of the angle, where the rotation was mentioned only briefly, as school mathematical language with few notions from scholarly mathematical language. Mary's explanation included gestures showing the angle as a measurement of a rotation, which was not possible in the textbook's brief reference to rotation. Consequently, the gestures contributed to highlight the rotation in the description of angles, as compared to the textbook, making the rationale for the measurement of angles a little deeper, positioning it closer to theory in the technology/theory box, than the textbook's explanation, which did not emphasise rotation. This result must be seen in the context of where it was taking place, where a deeper level of rationale in a grade five classroom was interpreted to include a language drawing from scholarly mathematical notions or explanations using more general systems. Hence, when Mary drew on the notion of limits in her explanation, she expressed a deeper level of rationale than showing examples of acute and obtuse angles, or simply stating that they are bigger or smaller than 90°.

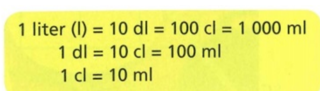
In summary, Figure 9 displays a version of praxeologies, where there is a match between the tasks but not fully between techniques or technology/theory.

Textbook praxis		*Classroom praxis*	
Task	*Technique*	*Task*	*Technique*
How to measure an angle	Measuring an angle with a protractor.	How to measure an angle	Measuring an angle with a protractor, and decide what scale to use on the protractor through a comparison with a right angle.
Textbook logos		*Classroom logos*	
Technology/Theory		*Technology/Theory*	
An angle is defined as two rays meeting in a common point, but also by naming the parts of an angle.		An angle is defined by naming the parts of an angle. The unit degrees is a measurement of a	

The unit degrees is a measurement of a rotation. Acute and obtuse angles are described to be respectively smaller or bigger than 90°.	rotation. The scope of acute and obtuse angles is described. Acute angles are described to go from zero towards 90° and obtuse angles are described to go from 90° and up.

Figure 9. Praxeologies for how to measure an angle.

Among the 15 praxeologies that were analysed, most were partly matched like the one described in the present article. The most common match was that the tasks were the same, and the teacher added techniques and/or technology compared to the textbook. Another type of comparison was a total match, when the teacher described the exact same technique and technology, with the same depth as the textbook. This type was not very common. A third type was a total mismatch. When this was the case, Mary used similar, but more complex tasks than we could find in the textbook. One example of differences in technology in such a comparison was the simple statements in the textbook compared to a general structure in Mary's teaching practice. For example, on the topic of measurement, the textbook offered the students a list of facts (see Figure 10), while Mary, instead drew from the SI-system of units, as well as from more familiar quantities, like length. Mary described how the same prefixes are used also to express volume in


1 liter (l) = 10 dl = 100 cl = 1 000 ml
1 dl = 10 cl = 100 ml
1 cl = 10 ml

litres.

Figure 10. Yellow box with facts about volume units, from the textbook.

From the explanations above, we inferred that the language Mary used drew on general structures, like those of scholarly mathematics. On the other hand, the language in the textbook, where facts about the units were stated, was inferred to be of a school mathematical character. Common to all differences in technologies/theories was that the language in Mary's explanations drew more from scholarly mathematics as compared to the textbook.

Discussion

In this article, our aim was to explore how praxeology can be used as an analytical framework for the comparison of a mathematics teacher's practice (as realised in the classroom) and communication in mathematics textbooks. One conclusion is that the adoption of praxeology may be a way to illuminate differences and similarities between communications in different contexts. Comparing technology/theory in classroom communication with technology/theory in a textbook, and likewise for techniques and tasks, enabled a deep analysis and comparison revealing differences in technologies where, as shown in this article, the teacher drew more from general structures in her explanations than was the case in the textbook.

We already know that the use of mathematics textbooks can influence mathematics teaching. Previous research tells us how classroom practice match textbooks in terms of the organisation of mathematics content or how time is spent throughout mathematics lessons (Barr, 1988; Haggarty & Pepin, 2002; Lepik et al., 2015). Compared to these studies the teaching practice in this study could be seen as closely related to the content of the textbook. The organisation of the content was very similar in all construed praxeologies. Other studies focus on the specificity of the communication in textbooks (e.g., Jablonka & Johansson, 2010; Jablonka et al., 2016). However, there is little research on detailed comparisons of the communication in textbooks, as compared to the classroom practices, in which they are used. In this article, we offer a way to compare the communication in terms of the tasks, the communicated techniques, and the technology/theory. We see this methodology as one way to focus on both micro and macro perspectives as described by Whittington (2006). Through this methodology, the contexts around a teacher can be examined without losing the micro perspective, including the teacher's own communication. One possibility for further studies could be to focus more on how the communications differ rather than just the occurrence of differences per se. To use a textbook in the mathematics classroom, even if it is used as the main source of exercises, does not necessarily predetermine what is taught and/or how it is taught. The use of praxeology in our analysis resulted in details becoming visible and hence possible to compare. Such details have been described in studies of textbooks as texts in themselves (Grave & Pepin, 2015; Haggarty & Pepin, 2002; Herbel-Eisenmann, 2007; Österholm & Bergqvist, 2013), but have not typically been directly compared with teachers' practices. This comparison is essential if we want to say something about the use of mathematics textbooks and teachers' practices in relation to how they communicate mathematics. Many studies claim a direct and extensive presence of the textbook (e.g., Johansson, 2006; Sosniak & Stodolsky, 1993), but more can be done to analyse how this presence manifests itself. The findings of this article may offer a starting point.

The choice to combine theory and technology as one unit was fruitful in our study. The communication in a grade five classroom is not very specific so it was convenient to see technology/theory as one unit. Seeing the technology/theory box as a continuum became a way to compare what communication was privileged in the discursive environment of teaching concepts. Most common was a privileging of school mathematical language where many definitions were taken for granted, but sometimes there were traces in the classroom communication of a scholarly mathematical language, closer to what Klisinska (2009) described as a specialised language, rich in definitions. This could, by extension, imply that teachers are also involved in external transposition, something Chevallard (1991) and Bosch et al. (2005) describe as being an activity of the noosphere (in this study, the textbook). In this study, it is rather the teacher, who draws from scholarly mathematics, more than what was possible to infer from the mathematics textbook.

We acknowledge that a single case study with only one teacher has many limitations. We, however, construe this case as a telling case (Andrews, 2016a; Mitchell, 1984), in that the teacher's practice illuminated various ways of communicating mathematical concepts. Hence, we have been able to present findings which illuminate how praxeologies may work as a framework for the comparison of the communication in mathematics classrooms and in textbooks. That being said, textbooks represent only one source for a teacher to determine what mathematics should be taught and how. There are several other sources of information, for example the national curriculum or in the professional literature for teachers. Adopted in a similar way to this article, praxeology would work well as a framework to compare the different sources with a teacher's classroom communication. This would give deeper insight into the particularities of a teacher's transposition process.

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